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Electric field and stresses concentrations at the edge of parallel electrodes in piezoelectric ceramics

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Abstract

Linear theory of piezoelectricity is used to solve the problem of electric field concentrations of a pair of parallel electrodes arrayed in one plane. Hence, the layer tends to deform in a plane mode. Fourier transforms are successfully applied to reduce electro-elastic boundary value problem to the solutions of integral equations. The integral equations are solved theoretically and the analytic solutions are carried out. It is found that electric field and stresses concentrate in the vicinity of the electric edge. The stresses at the edge of electrodes are so high that increases the failure possibility of materials. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Piezoelectric ceramics; Electromechanical coupling; Stresses concentration

1. Introduction

Piezoelectric ceramics is very easy to be manufactured, and its characters are easy to be changed by modifying its structure or adding other ion. In the materials, stresses and electric charges are generated by the application of an electric field or mechanical load. Because of these characters, piezoelectric ceramics have been widely used for actuators and sensors. In these devices, electrodes are arrayed in space. During operation, stress concentration is resulted from electric field concentration at the tip of electrodes. As a result, it is important to analyze the electric field and stresses distribution at the edge of electrodes (Uchino, 1998; Rao and Sunar, 1994). Due to the anisotropic electro-elastic properties and the electro-mechanical coupling in piezoelectric materials, the analysis is complicated. Recently, many researchers have contributed to studying of the behavior of elastic and electric variables in the vicinity of a surface electrode attached to piezoelectric ceramics. For instances, a lot of numerical solutions have been presented by Yang and Suo (1994), Hao et al. (1996), Hom and Shankar (1996) and Shindo et al. (1997). The electro-elastic variables behave differently on changing the distance between electrodes. So it is necessary to analyze the electric field concentration of a pair of electrodes.

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In this paper, attention is focused on the static piezoelectric behavior in the vicinity of a pair of electrodes in one plane. Equations for elastic and electric variables are studied based on the linear theory of piezoelectricity. Fourier transforms are utilized to reduce electro-elastic boundary value problem to the solutions in state space. The stresses and electric displacements in the vicinity of the electrodes edge are analyzed. In addition, the influences of distance between the pair of electrodes on electric field and stress concentrations at the edge of electrodes are shown graphically.

2. Fundamental governing equations and boundary conditions

2.1. Fundamental theory

If the body force, free electric charge and body electric current are ignored the stress equilibrium equations yield the form:

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \end{cases} \tag{1}$$

Gauss equation is

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0 \tag{2}$$

The piezoelectric constitute equations (6 mm symmetry) are

$$\begin{cases} \sigma_x = c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \varphi}{\partial z} \\ \sigma_y = c_{12}\frac{\partial u}{\partial x} + c_{11}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \varphi}{\partial z} \\ \sigma_z = c_{13}\frac{\partial u}{\partial x} + c_{13}\frac{\partial v}{\partial y} + c_{33}\frac{\partial w}{\partial z} + e_{33}\frac{\partial \varphi}{\partial z} \\ \tau_{yz} = c_{44}(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial x}) + e_{15}\frac{\partial \varphi}{\partial y} \\ \tau_{xz} = c_{44}(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}) + e_{15}\frac{\partial \varphi}{\partial x} \\ \tau_{xy} = c_{66}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \end{cases} \tag{3}$$

$$\begin{cases} D_x = e_{15}(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}) - \varepsilon_{11}\frac{\partial \varphi}{\partial x} \\ D_y = e_{15}(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}) - \varepsilon_{11}\frac{\partial \varphi}{\partial y} \\ D_z = e_{31}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + e_{33}\frac{\partial w}{\partial z} - \varepsilon_{33}\frac{\partial \varphi}{\partial z} \end{cases} \tag{4}$$

where u, v, w are the displacements, φ is electric potential, c_{ij} are elastic constants, e_{ij} are piezoelectric constants, ε_{ij} are the dielectric constants, D_x, D_y and D_z stand for electric displacement components.

2.2. Boundary conditions and assumptions

Using Cartesian coordinates, a piezoelectric ceramics with a pair of parallel electrodes showed in Fig. 1 is studied. In Fig. 1, the height between the pair of electrodes is $2h$, and the length of the electrode is $2a$.

We can consider the problem for plane strain. The displacements and electric potential become $u = u(x,z)$, $v = \text{constant}$, $w = w(x,z)$, $\varphi = \varphi(x,z)$. The equilibrium equations are simplified to

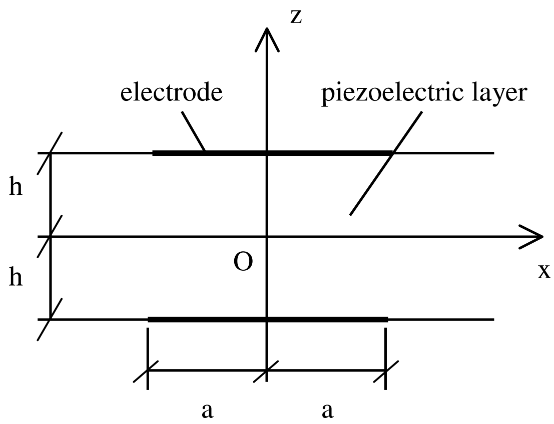


Fig. 1. The piezoelectric material with a pair of electrodes.

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0, \qquad \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} = 0 \tag{5}$$

Then Gauss equation is in the form of

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} = 0 \tag{6}$$

The constitutive equations are obtained as

$$\begin{cases} \sigma_x = c_{11}\frac{\partial u}{\partial x} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \varphi}{\partial z} \\ \sigma_y = c_{12}\frac{\partial u}{\partial x} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \varphi}{\partial z} \\ \sigma_z = c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial w}{\partial z} + e_{33}\frac{\partial \varphi}{\partial z} \\ \tau_{xz} = c_{44}(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}) + e_{15}\frac{\partial \varphi}{\partial x} \end{cases} \tag{7}$$

$$\begin{cases} D_x = e_{15}(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}) - \varepsilon_{11}\frac{\partial \varphi}{\partial x} \\ D_z = e_{31}\frac{\partial u}{\partial x} + e_{33}\frac{\partial w}{\partial z} - \varepsilon_{33}\frac{\partial \varphi}{\partial z} \end{cases} \tag{8}$$

If using displacements and electric potential as governing variables the governing equations are

$$\begin{cases} c_{11}\frac{\partial^2 u}{\partial x^2} + c_{44}\frac{\partial^2 u}{\partial z^2} + (c_{13} + c_{44})\frac{\partial^2 w}{\partial x\,\partial z} + (e_{31} + e_{15})\frac{\partial^2 \varphi}{\partial x\,\partial z} = 0 \\ (c_{13} + c_{44})\frac{\partial^2 u}{\partial x\,\partial z} + c_{44}\frac{\partial^2 w}{\partial x^2} + c_{44}\frac{\partial^2 w}{\partial z^2} + e_{15}\frac{\partial^2 \varphi}{\partial x^2} + e_{33}\frac{\partial^2 \varphi}{\partial z^2} = 0 \\ (e_{15} + e_{31})\frac{\partial^2 u}{\partial x\,\partial z} + e_{15}\frac{\partial^2 w}{\partial x^2} + e_{33}\frac{\partial^2 w}{\partial z^2} - \varepsilon_{11}\frac{\partial^2 \varphi}{\partial x^2} - \varepsilon_{33}\frac{\partial^2 \varphi}{\partial z^2} = 0 \end{cases} \tag{9}$$

Ignoring the rigidity of electrodes, the boundary conditions are written as

$$\varphi(x, \pm h) = \Phi^{\pm} \quad (|x| \leqslant a) \tag{10}$$

$$D_z(x, \pm h) = 0 \quad (|x| > a) \tag{11}$$

$$\sigma_z(x, \pm h) = 0 \quad (-\infty < x < +\infty) \tag{12}$$

$$\tau_{xz}(x, \pm h) = 0 \quad (-\infty < x < +\infty) \tag{13}$$

$$\sigma_x(x,z) = 0 \quad (|x| \to \infty) \tag{14}$$

3. Solution procedure

It should be noted that the coordinates are symmetrical. We only consider the part of $0 < x < \infty$. By using Fourier transforms, the displacements and electric potential can be expressed as

$$\begin{aligned}
\bar{u}(p,z) &= \sqrt{\frac{2}{\pi}}\int_0^\infty u(x,z)\sin(px)\,\mathrm{d}x \\
\bar{w}(p,z) &= \sqrt{\frac{2}{\pi}}\int_0^\infty w(x,z)\cos(px)\,\mathrm{d}x \\
\bar{\varphi}(p,z) &= \sqrt{\frac{2}{\pi}}\int_0^\infty \varphi(x,z)\cos(px)\,\mathrm{d}x
\end{aligned} \tag{15}$$

The Fourier reverse transforms are

$$\begin{aligned}
u(x,z) &= \sqrt{\frac{2}{\pi}}\int_0^\infty \bar{u}(p,z)\sin(px)\,\mathrm{d}p \\
w(x,z) &= \sqrt{\frac{2}{\pi}}\int_0^\infty \bar{w}(p,z)\cos(px)\,\mathrm{d}p \\
\varphi(x,z) &= \sqrt{\frac{2}{\pi}}\int_0^\infty \bar{\varphi}(p,z)\cos(px)\,\mathrm{d}p
\end{aligned} \tag{16}$$

Using the above notations, the governing equations are transformed to

$$\begin{aligned}
&c_{44}\frac{\partial^2\bar{u}}{\partial z^2} - c_{11}p^2\bar{u} - p(c_{13}+c_{44})\frac{\partial\bar{w}}{\partial z} - p(e_{31}+e_{15})\frac{\partial\bar{\varphi}}{\partial z} = 0 \\
&-p(c_{13}+c_{44})\frac{\partial\bar{u}}{\partial z} - p^2c_{44}\bar{w} + c_{44}\frac{\partial^2\bar{w}}{\partial z^2} - p^2e_{15}\bar{\varphi} + e_{33}\frac{\partial^2\bar{\varphi}}{\partial z^2} = 0 \\
&-p(e_{15}+e_{31})\frac{\partial\bar{u}}{\partial z} - p^2e_{15}\bar{w} + e_{33}\frac{\partial^2\bar{w}}{\partial z^2} + p^2\varepsilon_{11}\bar{\varphi} - \varepsilon_{33}\frac{\partial^2\bar{\varphi}}{\partial z^2} = 0
\end{aligned} \tag{17}$$

The equation can be written in state space as

$$\frac{\partial X}{\partial z} = \Pi X \tag{18}$$

in which

$$X = \begin{Bmatrix} \bar{u} \\ \bar{w} \\ \bar{\varphi} \\ \frac{\partial\bar{u}}{\partial z} \\ \frac{\partial\bar{w}}{\partial z} \\ \frac{\partial\bar{\varphi}}{\partial z} \end{Bmatrix} \qquad \Pi = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ k_1p^2 & 0 & 0 & 0 & k_2p & k_3p \\ 0 & l_2p^2 & l_3p^2 & l_1p & 0 & 0 \\ 0 & m_2p^2 & m_3p^2 & m_1p & 0 & 0 \end{bmatrix} \tag{19}$$

where,

$$
\begin{aligned}
k_1 &= c_{11}/c_{44}, \qquad k_2 = (c_{13}+c_{44})/c_{44}, \qquad k_3 = (e_{31}+e_{15})/c_{44} \\
l_1 &= [\varepsilon_{33}(c_{13}+c_{44}) + e_{31}(e_{15}+e_{31})]/(c_{44}\varepsilon_{33}+e_{31}e_{33}) \\
l_2 &= (c_{44}\varepsilon_{33}+e_{15}e_{31})/(c_{44}\varepsilon_{33}+e_{31}e_{33}) \\
l_3 &= (\varepsilon_{33}e_{15}-\varepsilon_{11}e_{31})/(c_{44}\varepsilon_{33}+e_{31}e_{33}) \\
m_1 &= [e_{33}(c_{13}+c_{44}) - c_{44}(e_{15}+e_{31})]/(e_{31}e_{33}+\varepsilon_{33}c_{44}) \\
m_2 &= (e_{33}c_{44}-e_{15}c_{44})/(e_{31}e_{33}+\varepsilon_{33}c_{44}) \\
m_3 &= (e_{15}e_{33}+c_{44}\varepsilon_{11})/(e_{31}e_{33}+\varepsilon_{33}c_{44})
\end{aligned}
\tag{20}
$$

The eigenvalue equation is

$$
\begin{vmatrix}
-\lambda & 0 & 0 & 1 & 0 & 0 \\
0 & -\lambda & 0 & 0 & 1 & 0 \\
0 & 0 & -\lambda & 0 & 0 & 1 \\
k_1p^2 & 0 & 0 & -\lambda & k_2p & k_3p \\
0 & l_2p^2 & l_3p^2 & l_1p & -\lambda & 0 \\
0 & m_2p^2 & m_3p^2 & m_1p & 0 & -\lambda
\end{vmatrix} = 0
\tag{21}
$$

Eq. (21) can be written as the following expression

$$
\lambda^6 + I_1p^2\lambda^4 + I_2p^4\lambda^2 + I_3p^6 = 0
\tag{22}
$$

in which,

$$
\begin{aligned}
I_1 &= k_2l_1 + k_3m_1 - k_1 - l_2 - m_3 \\
I_2 &= k_2(l_3m_1 - l_1m_3) + k_3(l_1m_2 - l_2m_1) + l_2m_3 - l_3m_2 + k_1m_3 + k_1l_2 \\
I_3 &= k_1(l_3m_2 - l_2m_3)
\end{aligned}
\tag{23}
$$

The solutions of Eq. (17) can be given by solving the eigenvalues of Eq. (22)

$$
\begin{aligned}
\bar{u}(p,z) &= \sum_j A_j e^{\lambda_j z} \\
\bar{w}(p,z) &= \sum_j B_j e^{\lambda_j z} \\
\bar{\varphi}(p,z) &= \sum_j C_j e^{\lambda_j z}
\end{aligned}
\tag{24}
$$

in which, $\lambda_j(p)$ is calculated from Eq. (22). Substituting Eq. (24) into boundary conditions Eqs. (12) and (13) yields

$$
\sum_j (c_{13}pA_j + c_{33}B_j\lambda_j + e_{33}C_j\lambda_j) = 0
\tag{25}
$$

$$
\sum_j [c_{44}(A_j\lambda_j - pB_j) - e_{15}pC_j] = 0
\tag{26}
$$

Then, A_j and B_j can be expressed by C_j

$$A_j = -\frac{(-c_{33}e_{15}\lambda_j + c_{44}e_{33}\lambda_j)pC_j}{c_{44}(c_{33}\lambda_j^2 + c_{13}p^2)}$$
$$B_j = -\frac{(c_{44}e_{33}\lambda_j^2 + c_{13}e_{15}p^2)C_j}{c_{44}(c_{33}\lambda_j^2 + c_{13}p^2)} \tag{27}$$

And boundary condition (11) can be written as

$$\int_0^\infty \sum_j f_j(p)C_j e^{\lambda_j h} \cos(px)\,dp = 0 \quad (|x| > a) \tag{28}$$

where

$$\begin{aligned} f_j(p)C_j &= (e_{31}pA_j + e_{33}B_j\lambda_j - \varepsilon_{33}C_j\lambda_j)/C_j \\ &= -\frac{c_{33}e_{15}e_{31}e_{33}\lambda_j^3p^2}{c_{44}(c_{33}\lambda_j^2 + c_{13}p^2)^2} + \frac{e_{31}e_{33}^2\lambda_j^3p^2}{(c_{33}\lambda_j^2 + c_{13}p^2)^2} - \frac{c_{13}c_{33}e_{15}^2e_{31}\lambda_jp^4}{c_{44}^2(c_{33}\lambda_j^2 + c_{13}p^2)^2} + \frac{c_{13}e_{15}e_{31}e_{33}\lambda_jp^4}{c_{44}(c_{33}\lambda_j^2 + c_{13}p^2)^2} \\ &\quad - \frac{c_{33}e_{15}e_{33}\lambda_j^2p}{c_{44}(c_{33}\lambda_j^2 + c_{13}p^2)^2} - \frac{e_{33}^2\lambda_j^2p}{c_{33}\lambda_j^2 + c_{13}p^2} + \frac{e_{33}\varepsilon_{33}\lambda_j^3}{c_{33}\lambda_j^2 + c_{13}p^2} - \frac{c_{13}e_{15}\varepsilon_{33}\lambda_jp^2}{c_{44}(c_{33}\lambda_j^2 + c_{13}p^2)} \end{aligned} \tag{29}$$

Eq. (28) is identical to the following Eq. (30)

$$f_j(p)C_j e^{\lambda_j h} = \int_0^a g(t)J_0(pt)\,dt \quad (|x| > a) \tag{30}$$

in which, $g(t)$ is the function to be determined and $J_0(pt)$ is zero-order Bessel function. Boundary condition (10) can be expressed as

$$\sqrt{\frac{2}{\pi}}\int_0^\infty \sum_j C_j e^{\lambda_j h} \cos(px)\,dp = \frac{\Phi}{h} \quad (0 \leqslant x < a) \tag{31}$$

Upon the substitution of Eq. (30) and then integrating by part, Eq. (31) becomes

$$\sqrt{\frac{2}{\pi}}\int_0^a g(t)\left[\int_0^\infty \frac{1}{p\sum_j f_j(p)} J_0(pt)\sin(px)\,dp\right]dt = -\frac{\Phi}{h}x \quad (0 \leqslant x < a) \tag{32}$$

or

$$\sqrt{\frac{2}{\pi}}\int_0^x \frac{g(t)\,dt}{\sqrt{x^2 - t^2}} + \sqrt{\frac{2}{\pi}}\int_0^a g(t)\left[\int_0^\infty m(p)J_0(pt)\sin(px)\,dp\right]dt = -\frac{\Phi}{h}x \tag{33}$$

where $m(p) = 1/p\sum_j f_j(p)^{-1}$.

Further, applying inverse Abel transformation of the first kind to both sides of Eq. (33), it yields

$$g(t) + t\int_0^a g(u)F(t,u)\,du = -\sqrt{\frac{\pi}{2}}\frac{\Phi}{h}t \tag{34}$$

in which the kernel function $F(t,u)$ is given by

$$F(t,u) = \int_0^\infty pm(p)J_0(pu)J_0(pt)\,\mathrm{d}p \tag{35}$$

The study of the present paper is limited to the case where the ratio of a/h is small. By using the expression of the Bessel function in power series form, $F(t,u)$ can be given. The solution of Eq. (34) can be obtained by the following iteration,

$$g_0(t) = \sqrt{\frac{\pi}{2}}\frac{\Phi}{h}t, \qquad g_{r+1}(t) = \sqrt{\frac{\pi}{2}}\frac{\Phi}{h}t - t\int_0^a g_r(t)F(t,u)\,\mathrm{d}u \quad (r = 0, 1, \ldots) \tag{36}$$

$g(t)$ is given by iteration up to $(a/h)^2$. Then, A_j, B_j, C_j, the electric field and displacement can be calculated.

4. Discussion of results

To evaluate the influence of distance between the electrodes, numerical calculations are carried out, the elastic constant c_{11} is 16.6×10^{10} N m^{-2}, c_{12} is 7.7×10^{10} N m^{-2}, c_{13} is 7.8×10^{10} N m^{-2}, c_{33} is 16.2×10^{10} N m^{-2}, c_{44} is 4.3×10^{10} N m^{-2}, the piezoelectric constant e_{31} is -4.4 C m^{-2}, e_{33} is 19.6 C m^{-2}, e_{15} is 11.6 C m^{-2}, the dielectric constant k_{11} is 1.12×10^{-8} C V^{-1} m^{-1} and k_{33} is 1.26×10^{-8} C V^{-1} m^{-1} (Nye, 1976). Figs. 2 and 3 show the distribution of $u(x,z)/(\phi/h)$ and $w(x,z)/(\phi/h)$ for $a/h = 1/10$. It can be seen that

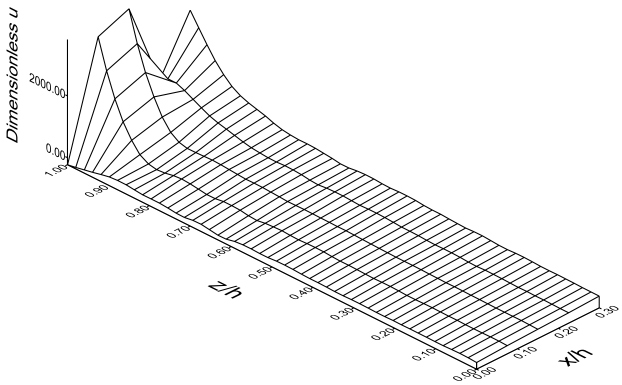


Fig. 2. Distribution of displacement u.

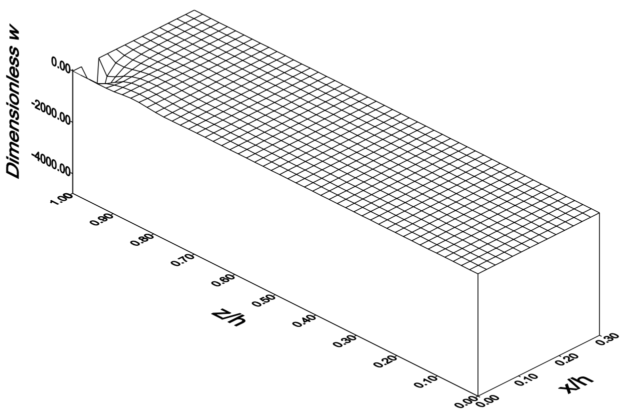


Fig. 3. Distribution of displacement w.

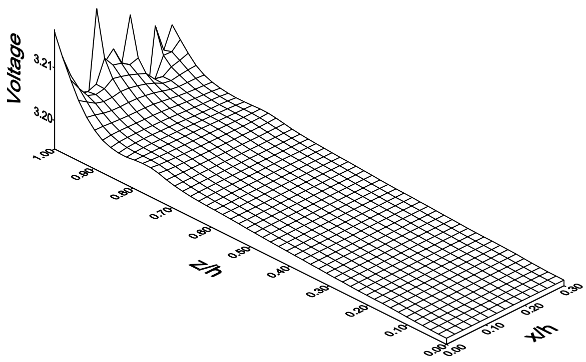


Fig. 4. Distribution of electric potential $\varphi(x,z)/(\Phi/h)$.

displacements near the electrode decrease acutely when z/h decreases. The displacements of $u(x,z)/(\phi/h)$ and $w(x,z)/(\phi/h)$ change abruptly at the tip of the electrodes and change lentamente in middle of the electrodes. The distribution of $\varphi(x,z)/(\phi/h)$ is shown in Fig. 4 for $a/h = 0.1$. The similar results can be given from the Fig. 4.

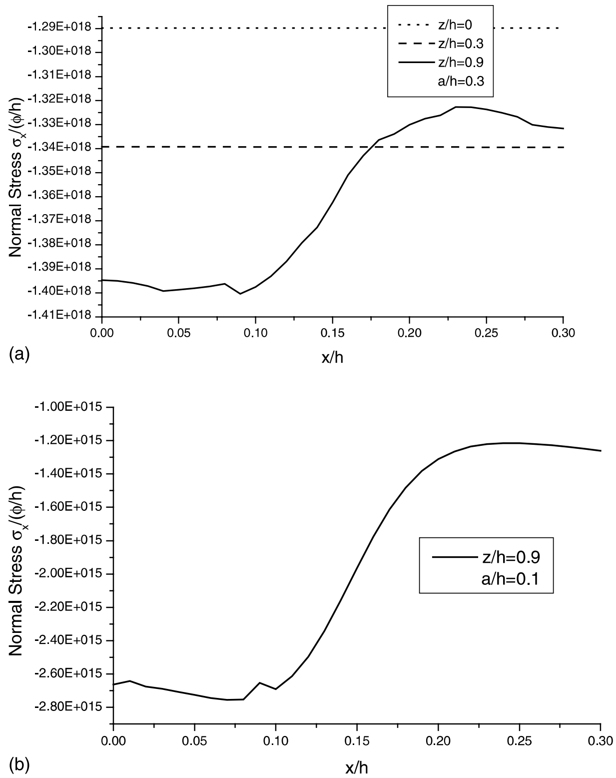


Fig. 5. Distribution of normal stress $\sigma_x/(\Phi/h)$ with x/h.

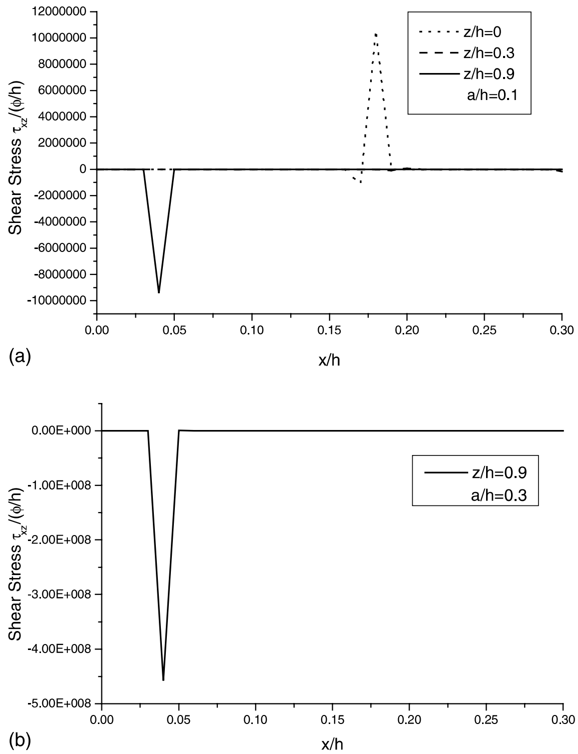


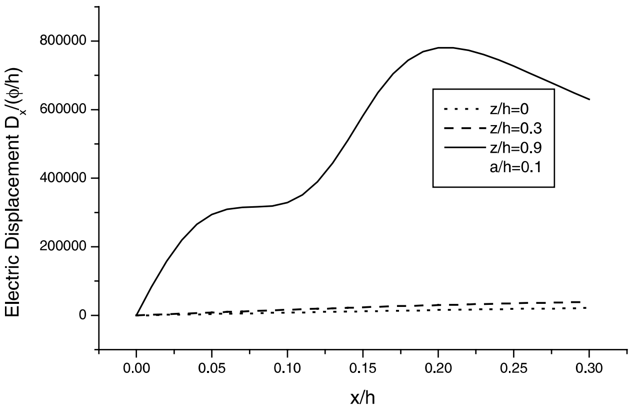
Fig. 6. Distribution of shear stress $\tau_{xz}/(\Phi/h)$ with x/h.

The distribution of stresses $\sigma_x/(\Phi/h)$ and $\tau_{xz}/(\Phi/h)$ are shown in Figs. 5 and 6. It can be found that the normal stress $\sigma_x/(\Phi/h)$ changes abruptly at the tip of the electrodes (e.g. $x/h = 0.1$, $a/h = 0.1$). Then its absolute value decreases gradually while x/h increases. It is found from Fig. 6b that the shear stress $\tau_{xz}/(\Phi/h)$ has a jump at the place where x/a is near to $1/2$.

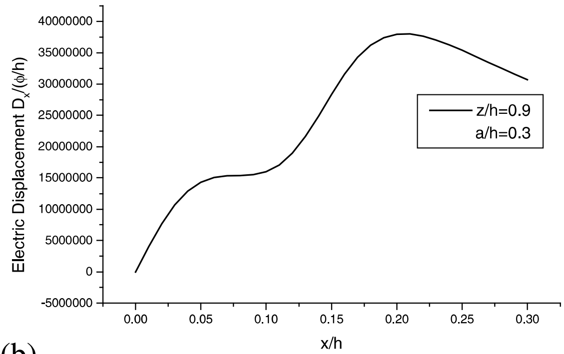
The distribution of dimensionless electric displacement $D_x/(\phi/h)$ and $D_z/(\phi/h)$ are shown in Figs. 7 and 8. There is point of inflection at the tip of electrode (e.g. $x/h = 0.1$, $a/h = 0.1$). This leads to electric displacement a sudden change of the edge of electrodes.

5. Conclusions

The distribution of the electric potential and displacement near the edge of electrodes are analyzed theoretically. The voltage and displacement change abruptly near the tip of the electrodes. The values of stresses change when distance between electrodes or distance between electrode and base changes. The electric field concentrates in the neighborhood of the electric edge. The values of stresses in vicinity of electrodes are so high as to give rise to the failure of materials.



(a)



(b)

Fig. 7. Distribution of dimensionless electric displacement D_x versus x/h.

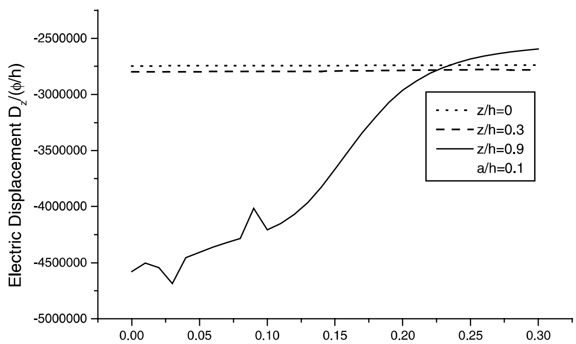


Fig. 8. Distribution of dimensionless electric displacement D_z versus x/h.

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